

# A Comedic Exploration of Algebraic Absurdities: Functorial Adventures in Infinite Dimensions

*(With Occasional Proofs and Existential Crises)*

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Indian Statistical Institute

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INDIAN STATISTICAL INSTITUTE

DOCTORAL THESIS

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**A Comedic Exploration of Algebraic  
Absurdities: Functorial Adventures in  
Infinite Dimensions**

*(With Occasional Proofs and Existential Crises)*

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# Abstract

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This LaTeX file was originally written while I was completing my PhD at the Indian Statistical Institute, Bangalore. After defending my thesis, I decided to convert it into a general-purpose template that can be used by anyone writing a thesis in LaTeX, and especially by ISI PhD scholars. The template includes a structured preamble, theorem environments, macros, color schemes, symbol and acronym management, and sample chapters.

It is designed to be flexible and easily adaptable, allowing users to focus on writing their thesis content without worrying about formatting. By following the examples in this template, scholars can create professional-looking theses with consistent styling, automated lists of symbols and acronyms, and well-formatted mathematical content.

This template aims to provide a practical starting point for LaTeX-based thesis writing while showcasing best practices for organizing large LaTeX projects.



# Dedication

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This template is dedicated to L<sup>A</sup>T<sub>E</sub>X, for faithfully not crashing (most of the time), for making my equations look nicer than my handwriting, and for being the silent companion of many sleepless thesis nights.

Also dedicated to anyone who has ever spent hours debugging a missing brace or forgotten `\end{document}`. May your compilations be fast and your error logs short!





# Acknowledgements

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First and foremost, I thank L<sup>A</sup>T<sub>E</sub>X for being a steadfast companion through countless hours of writing, compiling, and debugging. Without it, this template would probably still be a pile of loose notes and coffee stains.

I also extend my gratitude to spellcheckers, autocorrect, and online forums—especially Stack Exchange—for rescuing me from typos, mysterious compilation errors, and the occasional existential crisis caused by a missing brace.

Finally, thanks to anyone who ever glanced at this template and thought, “Hey, this might actually save me some time!” May your citations be complete, your figures properly aligned, and your bibliography entries forever consistent.

— *Indrajit Ghosh*



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# List of Symbols

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$\mathbb{N}$	Set of all natural numbers
$\chi(T)$	Characteristic projection of the closed operator $T$
$\mathbf{1}_\Omega$	Indicator function of the set $\Omega$
$\mathcal{M}_{\text{sa}}$	Self-adjoint part of the von Neumann algebra $\mathcal{M}$
$\mathcal{M}_{\text{proj}}$	Lattice of projections in $\mathcal{M}$
$\text{Aff}_s(\mathcal{M})$	Lattice of affiliated subspaces of $\mathcal{M}$
$\mathcal{M}_{\text{aff}}^{\text{MvN}}$	Murray–von Neumann affiliated operators associated with $\mathcal{M}$
$\mathcal{M}_{\text{aff}}^{\text{dd-c}}$	Densely-defined affiliated operators
$\mathcal{M}_{\text{aff}}$	All affiliated operators associated with $\mathcal{M}$
$\mathcal{M}_{\text{aff}}^c$	Affiliated operators that are closed
$\mathfrak{A}_{\text{aff}}$	Affiliated operators associated with a $C^*$ -algebra $\mathfrak{A}$
$\mathbb{Z}$	Set of all integers
$\mathbb{M}_n(\mathfrak{A})$	$n \times n$ matrices over a $C^*$ -algebra $\mathfrak{A}$
$\mathbf{C}^*\mathbf{Alg}$	Category of unital $C^*$ -algebras with unital $*$ -homomorphisms
$\mathbf{W}^*\mathbf{Alg}$	Category of von Neumann algebras with normal $*$ -homomorphisms
$S^{\mathbf{Kr}}$	Krein extension of the operator $S$
$S^{\mathbf{Fr}}$	Friedrich extension of the operator $S$
$\mathbb{R}$	Set of all real numbers
$\mathbb{C}$	Field of complex numbers
$\mathbb{M}_n(\mathbb{C})$	$n \times n$ matrix algebra over $\mathbb{C}$
$\mathcal{B}(\mathcal{H})$	Algebra of bounded operators on a Hilbert space $\mathcal{H}$
$\mathcal{C}(\mathcal{H})$	Closed unbounded operators on $\mathcal{H}$
$\text{Gr}(T)$	Graph of an operator $T$
$T^\dagger$	Kaufman inverse of operator $T$



# Introduction

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"Mathematics is serious, but  
your LaTeX doesn't have to be."

---

*Anonymous TeX Enthusiast*

Welcome to this templated PhD thesis! This chapter is your **\*\*user manual\*\***, showing how to wield the magical powers of LaTeX, AMS environments, colors, macros, symbols, and acronyms — all in one place.

## 1.1 Files in This Template

This template consists of several supporting files, each with a specific role:

- **main.tex:** The master document that ties everything together. Compile this to produce the final PDF.
- **preamble.tex:** Loads packages, defines theorem environments, AMS math structures, headers, and page styles.
- **colors.tex:** Defines all custom colors. Use them via `\textcolor{colorname}{text}` or in `hyperref`, `tikz`, and `tcolorbox`.
- **macros.tex:** Handy macros for math and text. Examples:
  - `\R`  $\Rightarrow \mathbb{R}$ , `\C`  $\Rightarrow \mathbb{C}$
  - `\Bh`  $\Rightarrow \mathcal{B}(\mathcal{H})$
  - `\tb{text}`  $\Rightarrow \text{text}$ , `\tn{text}`  $\Rightarrow$  normal text
  - Vector macros: `\bv`, `\bx`, `\btheta`, etc.
- **chapters/:** Contains separate `.tex` files for each chapter.
- **chapters/symbols.tex:** Defines all mathematical symbols for the glossary. Use them via `\gls{}`, e.g., [R](#).
- **chapters/acronyms.tex:** Defines all abbreviations. Use via `\ac{}`, e.g., Maximal Abelian Self-adjoint Algebras (MASA).

## 1.2 How to Use Theorem-Like Environments

Thanks to `preamble.tex`, you have access to **\*\*AMS theorem environments\*\*** and boxed theorems.

### Theorems, Lemmas, Propositions, Corollaries

- `\begin{theorem}[Optional Name] ... \end{theorem}`
- `\begin{lem}[Optional Name] ... \end{lem}`
- `\begin{prop}[Optional Name] ... \end{prop}`
- `\begin{cor}[Optional Name] ... \end{cor}`

### Definitions, Examples, Exercises

- `\begin{definition}[Optional Name] ... \end{definition}`
- `\begin{example}[Optional Name] ... \end{example}`
- `\begin{exercise}[Optional Name] ... \end{exercise}`

### Remarks, Notes, Cases, Notation

- `\begin{remark}[Optional Name] ... \end{remark}`
- `\begin{note} ... \end{note}` (unnumbered)
- `\begin{case}[Optional Name] ... \end{case}`
- `\begin{notation} ... \end{notation}`

### Boxed Theorems

`tcolorbox` provides visually appealing boxed theorems:

```
\begin{theo}{Name of Theorem}{label}  
  Statement goes here.  
\end{theo}
```

## 1.3 Symbols and Glossary

All symbols are defined in `chapters/symbols.tex`.

Use the command `\gls{}` to reference them. Examples:

- $\mathbb{N}$ : Set of natural numbers
- $\mathbb{R}$ : Set of real numbers
- $\mathcal{B}(\mathcal{H})$ : Algebra of bounded operators on a Hilbert space
- $\text{Aff}_s(\mathcal{M})$ : Lattice of affiliated subspaces of a von Neumann algebra
- $\mathcal{M}_{\text{aff}}^{\text{MvN}}$ : Murray–von Neumann affiliated operators

#### Tip

Always use `\gls{}` instead of typing symbols manually — this ensures they appear in the **List of Symbols**.



### 1.4 Acronyms

Defined in `chapters/acronyms.tex`.

Use `\ac{}` to reference them. Examples:

- MASA: Maximal Abelian Self-adjoint Algebra
- von Neumann Algebra (vNa): von Neumann Algebra
- Murray-von Neumann (MvN): Murray–von Neumann

#### Tip

Introduce acronyms at first use using `\ac{}`, then simply repeat using `\ac{}`. Your readers (and future self) will thank you.

### 1.5 AMS and Math Environments

All AMS environments from `amsmath`, `amssymb`, `mathtools` are available:

- Inline math: `$...$`
- Display math: `\[ ... \]`
- Alignments: `align`, `gather`, `multline`, `cases`
- Numbering: Equations are numbered per section (`\numberwithin{equation}{section}`)

### 1.6 Colors and Macros

- Colors: `colors.tex` provides `royalblue`, `indraRed`, `deepblue`, etc.
- Macros: `macros.tex` provides shortcuts for math and text. Examples:
  - Vectors: `\bv`, `\bx`, `\btheta`
  - Operators: `\grad`, `\curl`, `\div`
  - Algebras: `\Bh`, `\Ch`, `\aff{A}`
  - Bold or normal text: `\tb{text}`, `\tn{text}`

### 1.7 Compiling the Thesis

1. Always compile `main.tex`.
2. Make sure `preamble.tex`, `colors.tex`, and `macros.tex` are accessible via `\input{}`.
3. To generate the glossary and symbols:
  - `makeglossaries main`
  - `makeindex main`
4. View your PDF to check formatting, symbols, and theorem boxes.

## 1.8 Final Notes

This chapter is your **\*\*LaTeX cheat sheet\*\*** for this thesis template. Modify colors, macros, or environments freely — even serious mathematicians deserve a little fun.

# Category Theory and the Existential Crisis of Sets

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”Every set is lonely until  
someone defines a function from  
it.”

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*Anonymous Category Theorist*

## 2.1 Sets in Therapy

A set is just a collection of objects that never talk to each other. Category theory, in a surprising twist, gives them morphisms so they can at least make small talk.

**Definition 2.1.** A *category*  $\mathcal{C}$  is a structure consisting of:

- Objects (with commitment issues),
- Morphisms between them (sometimes called “maps of destiny”),
- Composition laws (that rarely commute with breakfast).

**Theorem 2.1** (Yoneda’s Existential Crisis). *Every object is completely defined by how others perceive it. Formally, for each  $A$  in  $\mathcal{C}$ ,*

$$\mathrm{Hom}_{\mathcal{C}}(-, A)$$

*contains all the gossip about  $A$ .*

*Proof.* This proof is left as an exercise in self-reflection. ■

*Remark 2.1.* Set theorists often call category theory “philosophy with diagrams.” Category theorists call set theory “philosophy without style.”



# The Spectrum Always Rings Twice: Adventures in Functional Analysis

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"Every operator has a spectrum;  
only some are worth listening  
to."

---

*Spectral Analyst, ca. 2025*

## 3.1 Prelude to Spectral Analysis

Given a Hilbert space  $\mathcal{H}$  and a bounded operator  $T \in B(\mathcal{H})$ , the *spectrum*  $\sigma(T)$  is the set of complex numbers  $\lambda$  for which  $T - \lambda I$  is not invertible. In other words, it's where analysis stops and magic begins.

**Moral:** The resolvent set is like your to-do list — everything outside it is under control.

**Theorem 3.1** (Spectral Decomposition, Coffee Edition). *Every normal operator  $T$  can be written as*

$$T = \int_{\sigma(T)} \lambda dE(\lambda)$$

where  $E$  is a projection-valued measure, and the integral is taken with sufficient caffeine.

*Proof.* The proof involves standard techniques from measure theory and late-night panic. ■

**Example 3.1.** If  $T$  is self-adjoint,  $\sigma(T) \subset \mathbb{R}$ . If  $T$  is a PhD student,  $\sigma(T)$  includes several imaginary parts.



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